OPTIMIZATION IN CRASHING PROJECT SCHEDULE WITH METHODS SELECTION

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ABSTRACT
Crashing is a process of expediting project schedule by compressing the total project duration. It is helpful both in executing process, when managers want to anticipate the late project status, and in planning process, when the initial estimation does not meet objective time completion’s key stakeholder. Crashing the schedule plan would not only affect the duration of certain task but also the method applied to it indirectly. All methods that can complete the task could have different cost slope. In this paper we want to present a model to select the best method of crashed task to meet minimum cost. Binary Integer Nonlinear programming is used in this paper. The output of this model will determine task to crash, time to reduce, method, and the minimum cost of project.

Key words: Crashing, Cost Slope, Integer Nonlinear Programming, Optimization

1. INTRODUCTION
Projects comprise a series of activities, some of which must be performed sequentially and others that can be performed in parallel with other activities. This collection of series and parallel tasks can be modeled as a network [2].

Steps in the Schedule Planning Process involve the following steps: 1) Define activities; 2) Sequence activities; 3) Estimate activity resources; 4) Estimate activity durations; 5) Develop Schedule. [6] But it needs 3 more steps to crash the schedule, the steps are: 6) Determine the critical path. 7) Determine the cost slope of critical task. 8) Crash the schedule.

Moreover, current crashing method copes with single method for each critical activity. This paper wants to present a model to solve the problem that each task can be done by more than one method. Thus it does not only determine the amount of time to reduce to meet minimum crashing cost, but also which method will be used for.

Since the process mapping in project management using the Network Diagram, the task that can be done by more than one method (the "OR" condition) cannot be seen. The schedule case study will be depicted by using the Functional Flow Block Diagram (FFBD), this condition is illustrated in Figure 2. Figure 1 depicts the project schedule case study using Activity On Arc and Activity On Node.

![Graphical representation of project schedule](image-url)
Figure 1. Example project schedule using (a) Activity on Arc; (b) Activity on Node

In Figure 1 can be seen that if a task can be done by more than one method, the Network Diagram cannot accommodate this problem. There are several models that can describe this condition such as IDEF3, FFBD, but we chose to describe it using FFBD. Figure 2 describes the condition.

Figure 2. FFBD for Project Schedule Case Study

It can be seen in Figure 2 that the cost of an activity may vary depending on the duration of A and methods used. This problem can be solved analytically by making a Integer Nonlinear Programming Model, where it takes two decision variables, the first decision variable to choose the amount of time will be reduced and the second is a variable to select the method that will be used by the lowest cost objective function.

So, this paper wants to answer several questions as follow:

“Which activities have to crash and what method has to use to meet minimum project cost with targeted project time completion?”

“Which activities have to crash and what method has to use to meet minimum project time completion with available fund?”

2. LITERATURE REVIEW

2.1. The General Linear Programming Model of CPM/PERT Network

The solution of this linear programming model will indicate the earliest time of each node in the network and the project duration [3].

\[ \text{Min } Z = X_n \]  \hspace{1cm} (1)

Subject to
\[ X_i - X_j - t_{ij} \geq 0 \]
\[ X_i, X_j \geq 0 \]

Where
\[ X_i = \text{earliest event time of node } i. \]
\[ X_j = \text{earliest event time of node } j. \]
\[ t_{ij} = \text{time of activity } i \rightarrow j. \]
\[ n = \text{number of the last node in the network}. \]

2.2. Linear Programming Model of Crashing

The objective function is to reduce the project duration at the minimum possible crash cost. The objective function coefficients are the activity cost slope, the variable \( y_{ij} \) indicate the number of time each activity will be reduced [1][2][3][4].

\[ \text{Min } Z = \sum_i \sum_j CS_{ij} \cdot Y_{ij} \]  \hspace{1cm} (2)

Subject to
\[ Y_{ij} \leq r_{ij} \quad \text{for all activities } i \rightarrow j \]
\[ X_i + t_{ij} - Y_{ij} \geq X_j \quad \text{for all activities } i \rightarrow j \]
\[ X_n \leq T \]
\[ X_i, X_j, Y_{ij} \geq 0 \]

Where
\[ X_i = \text{earliest start event time of node } i. \]
\[ X_j = \text{earliest finish event time of node } j. \]
\[ t_{ij} = \text{time of activity } i \rightarrow j. \]
\[ n = \text{number of the last node in the network}. \]
\[ r_{ij} = \text{number of the range amount activity } i \rightarrow j \]
\[ \text{can be crashed.} \]
\[ T = \text{Targeted project completion time}. \]
\[ CS_{ij} = \text{Cost Slope of activity } i \rightarrow j. \]

If we assume that the relationship between crash cost and crash time is linear, then an activity can be crash by $ cost slope per time.

Figure 3. Time-Cost Relationship
2.3. Functional Flow Block Diagram

A Functional Flow Block Diagram (FFBD) is a multi-tier, time-sequenced, step-by-step flow diagram of a system’s functional flow. The FFBD notation was developed in the 1950s, and is widely used in classical systems engineering. FFBDs are one of the classic business process modeling methodologies, along with flow charts, data flow diagrams, control flow diagrams, Gantt charts, PERT diagrams, and IDEF [7].

Figure 4. "AND" condition

Figure 5. "OR" condition

Figure 6. "AND" and "OR" condition in FFBD

2.4. Terminology

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPM</td>
<td>Critical Path Method</td>
</tr>
<tr>
<td>PERT</td>
<td>Program Evaluation and Review Technique</td>
</tr>
<tr>
<td>AOA</td>
<td>Activity on Arc</td>
</tr>
<tr>
<td>AON</td>
<td>Activity on Node</td>
</tr>
<tr>
<td>FFBD</td>
<td>Functional Flow Block Diagram</td>
</tr>
<tr>
<td>IDEF</td>
<td>Integrated Definition</td>
</tr>
</tbody>
</table>

3. METHODOLOGY

Problem that state in introduction section can be solved by using existing methodology. The steps are:

Step 1 - Define activities, The process of identifying the specific actions to be performed to produce the project deliverables.

Step 2 - Sequence activities, The process of identifying and documenting relationships among activities. Network diagram should be converted to FFBD in order to describe the alternatives method.

Step 3 - Estimate activity resources and Estimate activity durations, The process of estimating the type and quantities of material, people, equipment, and supplies required to perform each schedule activity and approximating the number of work periods needed to complete individual activities with estimated resources. This process is also determining normal time, normal cost, crash time, crash cost, and cost slope.

Step 4 - Develop Schedule, The process of analyzing activity sequences, durations, resource requirements, and schedule constraints to create the project schedule.

Step 5 - Determine the critical path, The critical path method calculates the theoretical early start and finish dates, and late start and finish dates, for all activities by performing a forward pass analysis and a backward pass analysis through the project schedule network paths. Activities on a critical path are called "critical activities." A critical path is normally characterized by zero total float/slack across the critical path and network paths can have multiple near critical paths.

Step 6 - Crash the schedule and choose the best method to accomplish the multi-method activities, The output of the model in this paper will determine task to crash, time to reduce, the best method to meet the minimum cost of project.

To develop the proposed model that describe in step 6, several scenario involving combination of serial parallel task are analyzed. Scenario 1 - project that consists of 2 serial and 2 parallel activities and its cost are illustrated in Figure 7 and 8.
Estimated minimum cost of task A in normal condition will be described below.

$$C_{12}(Y) = 50 . Y + 250 \quad 0 \leq Y \leq 4$$

$$C_{23}(Y) = 100 . Y + 100 \quad 0 \leq Y \leq 4$$

From Fig 8 it shows that the minimum cost in normal condition is $100 by using Method-2. To represent the “or” condition above in mathematical model we can put the binary variable, $H_{ij}$, in it.

$$C_{ij}(Y) = \min (C_{ij} + CS_{ij}, Y_{ij}). H_{ij} + (\min C_{ij} + CS_{ij}, Y_{ij}). H_{ij2}$$

After Run several experiment scenario we find that Equation above not only can calculate serial, and parallel, but also serial with “OR” condition. To cope with complex project, we have to redesign by adding the binary variables and constraints that can link the serial-parallel (combination) condition. The objective function is to reduce the project duration, and select the best method at the minimum possible project completion cost.

The objective function coefficients of the model are the activity cost slopes, the variable $y_{ij}$ indicates the number of time each activity will be reduced and binary variable $H_{ij}$ indicates the binary decision to select the best method to use. The proposed model in this paper is presented (4) below.

**The Optimization in Crashing Project Schedule with Methods Selection Model:**

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{a_{ij}=1}^{A_{ij}} \left( \min C_{ija} + CS_{ija}, Y_{ija} \right) . H_{ija}$$

Subject to

$$S_{a_{ij}=1}^{A_{ij}} H_{ija} = 1, \text{ for all activities } i \rightarrow j$$

$$Y_{ija} \leq r_{ij}, \text{ for all activities } i \rightarrow j$$

$$X_{i} + t_{ij} - Y_{ija} \geq X_{f}, \text{ for all activities } i \rightarrow j$$

$$X_{a} \leq T$$

$$X_{i}, X_{f}, Y_{ija} \geq 0$$

$$H_{ija} \text{ binary}$$

Where

- $X_{i}$ is earliest start event time of node $i$.
  
- $Y_{ija}$ is amount activity $i \rightarrow j$ can be crashed.
  
- $X_{f}$ is earliest finish event time of node $j$.
  
- $t_{ij}$ is time of activity $i \rightarrow j$.
  
- $n$ is number of the last node in the network.
  
- $r_{ij}$ is number of the range amount activity $i \rightarrow j$ can be crashed.
  
- $T =$ Targeted project completion time.
  
- $CS_{ija} =$ Cost Slope of activity $i \rightarrow j$.
  
- $C_{ija}$ is Cost of activity $i \rightarrow j$. 
\[ a_{ij} \] = Number of methods alternatives of activity \( i \rightarrow j \)
\[ = 1, 2, \ldots, A_{ij} \text{(If a Task has only a method the } A_{ij} \text{ will be 1)} \]

\[ C_{ij} = \text{Cost of activity } i \rightarrow j \text{ using method a.} \]
\[ C_{S ij} = \text{Cost Slope of activity } i \rightarrow j \text{ using method a.} \]
\[ H_{ij} = \text{Binary decision of using method a for activity } i \rightarrow j. \]

The solution of this integer binary programming model will indicate the earliest time of each node in the network, project cost, time to be reduced, and the method to be used.

4. CASE STUDY

To demonstrate the proposed approach, a case study involving serial, parallel, and alternatives methods activities combination is introduced in Table 1.

Table 1. Time, Method, and Cost for Case-Study

<table>
<thead>
<tr>
<th>Activity</th>
<th>( A_{ij} ) Method</th>
<th>Time (weeks)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Crash</td>
<td>Normal</td>
</tr>
</tbody>
</table>

**Problem-1:** in normal condition, organization used to complete this kind of project around 24 weeks. But the key stakeholder want the project has to be done in 15 weeks. Which activities have to crash and using what method to meet minimum project cost?

**Solve-1:** As it can be seen in Table 1 that activities, relationships, methods, time, and costs have been given thus Step1, Step2, and Step3 in methodology could be skipped. Step 4 and step 5 is illustrated in Figure 11 by documenting schedule using normal condition.

For Task A, there are three alternatives methods could be used using FFBD which it can be seen in Figure 2.

Finally (step 6), we must indicate each activity cost slope to build the objective’s model.

Consider that a cost slope is differentiated from the equation of its task cost line or cost function \( C_{ij} \).

Figure 12 and Table 2 depict the circumstance.

\[
CT_{ij}(t_{ij}) = 23500 - 1500t_{ij} \quad 5 \leq t_{ij} \leq 9
\]

Figure 12. Cost of time Function of Task A using Method-1

If the cost of time function as an activity like \( CT_{ij} = f(t_{ij}) \) where \( t_{ij} \) as working time an activity, then the rate of \( CT_{ij} \) is \( \frac{dCT_{ij}}{dt_{ij}} \). Then we know that the rate of A activity is \( \frac{dCT_{ij}}{dt_{ij}} = -1500 \). It means that the value of -1500 will decrease $1,500,000 if the working time of the project \( t_{ij} \) increases 1 week. If the cost of reduction time function is described as \( C_{ij} = f(Y_{ij}) \) where \( Y_{ij} \) as a symbol of time reducing an activity, then \( C_{ij} = \text{normal cost} + \left( -\frac{dCT_{ij}}{dt_{ij}} \right) \cdot Y_{ij} \). Figure 13 shows the illustration of accelerating cost of activity A:

Figure 11. Case Study’s Schedule (Network Diagram)

Figure 12. Cost of time Function of Task A using Method-1
Cost slope Activity A using method-1 is differentiated from accelerating cost of activity A
\[ (CS_{121}) = \left( \frac{dC}{dY_{12}} \right) = 1500. \]

The Resume can be seen in Table 2.

### Table 2. Cost of Reduction Time Function and Cost Slope for Each Activity

<table>
<thead>
<tr>
<th>Activity</th>
<th>Node</th>
<th>Cost of Reduction Time Function</th>
<th>Range</th>
<th>Cost Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>i=1,j=2</td>
<td>10000 + 1500 \cdot Y_{12}</td>
<td>0 \leq Y_{12} \leq 4</td>
<td>1500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8000 + 3000 \cdot Y_{12}</td>
<td>0 \leq Y_{12} \leq 4</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9000 + 1600 \cdot Y_{12}</td>
<td>0 \leq Y_{12} \leq 4</td>
<td>1600</td>
</tr>
<tr>
<td>B</td>
<td>i=1,j=3</td>
<td>4000 + 1000 \cdot Y_{13}</td>
<td>0 \leq Y_{13} \leq 2</td>
<td>1000</td>
</tr>
<tr>
<td>C</td>
<td>i=2,j=4</td>
<td>7500 + 1250 \cdot Y_{24}</td>
<td>0 \leq Y_{24} \leq 2</td>
<td>1250</td>
</tr>
<tr>
<td>D</td>
<td>i=3,j=5</td>
<td>14000 + 2500 \cdot Y_{35}</td>
<td>0 \leq Y_{35} \leq 3</td>
<td>2500</td>
</tr>
<tr>
<td>E</td>
<td>i=4,j=6</td>
<td>9000 + 9000 \cdot Y_{46}</td>
<td>0 \leq Y_{46} \leq 1</td>
<td>9000</td>
</tr>
<tr>
<td>F</td>
<td>i=5,j=7</td>
<td>1500 + 3000 \cdot Y_{57}</td>
<td>0 \leq Y_{57} \leq 1</td>
<td>3000</td>
</tr>
<tr>
<td>G</td>
<td>i=6,j=7</td>
<td>3500 + 1500 \cdot Y_{67}</td>
<td>0 \leq Y_{67} \leq 2</td>
<td>1500</td>
</tr>
</tbody>
</table>

The Optimization in Crashing Project Schedule with Methods Selection Model using LINGO 10:

The mathematical model as described before (methodology section) can be solved by LINGO 10 software program. This is possible if we write the objective function and all constraints with the role of RHS (all variables must be written in the left side and constant values in the right side of the symbol "="). Equation below shows the description of mathematical model for LINGO 10:

\[
\text{min} = (10000 + 1500 \cdot Y_{12}) \cdot H_{121} + (8000 + 3000 \cdot Y_{12}) \cdot H_{122} + (4000 + 1000 \cdot Y_{13}) + (7500 + 1250 \cdot Y_{24}) + (14000 + 2500 \cdot Y_{35}) + (9000 + 9000 \cdot Y_{46}) + (1000 + 3000 \cdot Y_{57}) + (1500 + 3000 \cdot Y_{67}) ;
\]

H_{121} + H_{122} + H_{123} = 1;

x_7 = 15;

x_1 = 0;

Y_{12} = 4;

Y_{13} = 2;

Y_{24} = 2;

Y_{35} = 3;

Y_{46} = 1;

Y_{57} = 1;

Y_{67} = 2;

x_2 - x_1 + Y_{12} = 9;

x_3 - x_1 + Y_{13} = 4;

x_4 - x_2 + Y_{24} = 5;

x_5 - x_3 + Y_{35} = 8;

x_6 - x_4 + Y_{46} = 3;

x_7 - x_5 + Y_{57} = 5;

x_7 - x_6 + Y_{67} = 7;

@bin(H_{121});

@bin(H_{122});

@bin(H_{123});

end

Solve-1 (LINGO 10):

Local optimal solution found.

Objective value: 71400.00

Extended solver steps: 0

Total solver iterations: 6

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y_{12}</td>
<td>4.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>H_{121}</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>H_{122}</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>H_{123}</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Y_{13}</td>
<td>2.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Y_{24}</td>
<td>2.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Y_{35}</td>
<td>2.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Y_{46}</td>
<td>2.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Y_{57}</td>
<td>2.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Y_{67}</td>
<td>2.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>X_7</td>
<td>15.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>X_1</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>X_2</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>X_3</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>X_4</td>
<td>8.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>X_5</td>
<td>10.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>X_6</td>
<td>10.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Row Slack or Surplus Dual Price
1 71400.00 -1.000000
2 0.000000 -15400.00
3 0.000000 10000.00
4 0.000000 -10000.00
5 0.000000 7400.00
6 0.000000 0.000000
7 0.000000 7750.00
8 3.000000 0.000000
9 0.000000 0.000000
10 0.000000 0.000000
11 0.000000 0.000000
12 0.000000 -9500.00
13 0.000000 -10000.00
14 0.000000 -9000.00
15 0.000000 0.000000
16 0.000000 0.000000
17 0.000000 0.000000
18 0.000000 0.000000
The values of the decision variables and constraints from LINGO’s report form problem-1 are:

\[ Y_{12} = 4, \ H_{121} = 0, \ H_{122} = 0, \ H_{123} = 1, \ Y_{13} = 2, \ Y_{24} = 2, \ Y_{35} = 0, \ Y_{46} = 1, \ Y_{57} = 0, \ Y_{67} = 2, \ X_1 = 0, \ X_2 = 5, \ X_3 = 2, \ X_4 = 9, \ X_5 = 10, \ X_6 = 10, \ X_7 = 15. \] And the value of the objective function (the total of normal + crashing cost) is $71,400.

It means to meet 15 weeks time completion we have to reduce Task A, B, C, E, G for 4, 2, 2, 1, and 2 days consecutively. And Task A has to use Method-3 to reach minimum project cost.

The experiments that had done using 10 scenarios from crashing the project for 1 day to maximum crashing the project (9 days) are showed in Table 3. Moreover the maximum target time completion was 15 days. 14 days of targeted project time completion will meet unfeasible condition. We can see all result including all variables and objective in Table 3.

### Table 3. Result of the experiments

<table>
<thead>
<tr>
<th>Task</th>
<th>$X_{23}$</th>
<th>$X_{34}$</th>
<th>$X_{45}$</th>
<th>$X_{56}$</th>
<th>$X_{67}$</th>
<th>$X_{78}$</th>
<th>$X_{97}$</th>
<th>$X_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>46750</td>
<td>50000</td>
<td>51650</td>
<td>53600</td>
<td>56600</td>
<td>57200</td>
<td>58600</td>
<td>66800</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>13</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>16</td>
<td>15</td>
<td>15</td>
<td>14</td>
<td>13</td>
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<tr>
<td>H</td>
<td>23</td>
<td>22</td>
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<td>20</td>
<td>19</td>
<td>18</td>
<td>17</td>
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</tr>
<tr>
<td>I</td>
<td>23</td>
<td>22</td>
<td>21</td>
<td>20</td>
<td>19</td>
<td>18</td>
<td>17</td>
<td>16</td>
</tr>
</tbody>
</table>

Based on the resume of result presented in Table 3, it is shown that if we want to crash the project for 1 and 2 days we should crash activity C for 1 and 2 days respectively. If we were seeing Figure 11 and Table 2 above it tells us that activity C was the lowest cost slope of critical activities.

Since the $Y$ for activity C range 0 to 2 days ($Y_{ac}$) make activity G has to crash 1 day and activity C maximally if we would finish the project in 21 day.

For Method that is used to activity A is also shown by Table 3. It is told that if the project has to crash for 1 to 4 days Method 2 will be the best method to use. Crashing the project for 5 to maximum crash (9 days) makes method 3 the best one. It is presented in Figure 15.

“How to optimize the time crashing project with available fund?” This question is also can be used to validate the model.

**Problem-2:** “How many days the optimal project time completion with $53,000 available fund?” The model solved using LINGO 10 is shown below.
\[ \begin{align*}
\text{min} &= X_7; \\
&= (10000 + 1500 \cdot Y_{12}) \cdot H_{121} \\
&+ (8000 + 3000 \cdot Y_{12}) \cdot H_{122} \\
&+ (9000 + 1600 \cdot Y_{12}) \cdot H_{123} \\
&+ (4000 + 1000 \cdot Y_{13}) \\
&+ (7500 + 1250 \cdot Y_{24}) \\
&+ (14000 + 2500 \cdot Y_{35}) \\
&+ (9000 + 9000 \cdot Y_{46}) \\
&+ (1500 + 3000 \cdot Y_{57}) \\
&+ (3500 + 1500 \cdot Y_{67}) \\
&= 53000; \\
H_{121} + H_{122} + H_{123} &= 1; \\
x_1 &= 0; \\
Y_{12} &= 4; \\
Y_{13} &= 2; \\
Y_{24} &= 2; \\
Y_{35} &= 3; \\
Y_{46} &= 1; \\
Y_{57} &= 1; \\
Y_{67} &= 2; \\
x_2 - x_1 + Y_{12} &= 9; \\
x_3 - x_1 + Y_{13} &= 4; \\
x_4 - x_2 + Y_{24} &= 5; \\
x_5 - x_3 + Y_{35} &= 8; \\
x_6 - x_4 + Y_{46} &= 3; \\
x_7 - x_5 + Y_{57} &= 5; \\
x_7 - x_6 + Y_{67} &= 7; \\
@ \text{bin}(H_{121}); \\
@ \text{bin}(H_{122}); \\
@ \text{bin}(H_{123}); \\
\end{align*} \]

Solve-2:

Local optimal solution found.

Objective value: 20.00000

Extended solver steps: 0

Total solver iterations: 22

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
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<tr>
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<td>(x_3)</td>
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<td>(x_4)</td>
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<td>(x_6)</td>
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</table>

The values of the decision variables and constraints from LINGO’s report form problem-2 are:

\(X_7=20\), \(Y_{12}=0\), \(H_{121}=0\), \(H_{122}=1\), \(H_{123}=0\), \(Y_{13}=0\), \(Y_{24}=2\), \(Y_{35}=0\), \(Y_{46}=0\), \(Y_{57}=0\), \(Y_{67}=2\), \(x_1=0\), \(x_2=9\), \(x_3=4\), \(x_4=12\), \(x_5=12\), \(x_6=15\). And the value of the objective function (the optimal project time completion with $53000 available funds) is 20 days. It means that we could complete the project for 20 days if we have $53,000 by crashing activities C and G for 2 days each activity. This identical result that can be seen in Table 3 shows the validation of this model to cope with problems in this paper.

### 5. IMPLEMENTATION

Model implementation covers a broad spectrum of decision making in planning and executing phase. The approach describe in this paper has been successfully implemented in solving selection problems. In project planning, we know that “or” conditions are very familiar. For example the condition for selecting vendor that usually using “make or buy analysis”. But the downside is the “make or buy analysis” selecting individually the alternative without concerning the schedule. This paper demonstrates that there is a relationship between cost and time. Thus, making the decision related to time or cost have to concern both factors.

### 6. CONCLUSIONS

Method selection in project is cannot characterized by existing process modeling, Network Diagram. Several modeling approaches could be used to describe this “or” condition, like IDEF0, and FFBD. In this paper Functional Flow Block Diagram is used to describe the conditions.

In this study, the novel “or” condition that consist of several alternatives methods combining the schedule planning has been proposed to optimize the project cost. Integer binary programming can be used to optimize the schedule, cost, and method selection simultaneously. The model demonstrates making decision by choosing the lowest cost slope of critical activities to reduce the project optimally. Regarding nonlinear form of the model, this research using LINGO 10 to solve the problems.

There are several additional research needs in implementing the model in executing and monitoring and controlling phase where the random/uncertainty condition will be faced. This random condition, like performance, needs deep research in simulation rather than analytical deterministic approaches.
7. REFERENCES